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Goods allocation by queuing and the occurrence of violence: A probabilistic analysis

Amitrajeet A. Batabyal [a,*], Gregory J. DeAngelo [b]

[a] Department of Economics, Rochester Institute of Technology, 92 Lomb Memorial Drive, Rochester, NY 14623-5604, USA
[b] Department of Economics, Rensselaer Polytechnic Institute, Russell Sage Laboratory, 110 8th Street, Troy, NY 12180, USA

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ABSTRACT

Goods for which demand greatly exceeds supply are frequently allocated to citizens using queuing mechanisms. However, violence can occur either when queues are very long or when large numbers of citizens are not provided goods being allocated with queuing mechanisms. Hence, we use the theory of discrete-time Markov chains (DTMCs) to construct and analyze models in which we explicitly account for queue length and the number of citizens who are not provided a good that is allocated with a queuing mechanism. Specifically, we first delineate a version of our DTMC model in which there is no capacity constraint. Second, we state two key properties of this model and then we derive our first metric of the likelihood of violence. Finally, we describe an alternate version of our DTMC model with a capacity constraint and then we derive our second metric of the likelihood of violence.

1. Introduction

There are many instances in developing countries and in transition economies where public officials use queuing mechanisms to allocate scarce goods to citizens. The most common reason for using a queuing mechanism is that the pertinent public official wishes to keep the price of the underlying good below its market clearing level. When this happens, there is an imbalance between demand and supply and hence some non-price mechanism must be used for the purpose of allocating the scarce good. Closely related to this idea is the notion that on occasion, public officials want to discriminate in favor of low income citizens who would otherwise be “priced out” of the market if market forces alone determined the price at which a scarce good is bought and sold. Because these low income citizens who are likely to be “priced out” of the market for a scarce good also typically have “low time costs” relative to higher income citizens with “high time costs,” a non-price mechanism such as a queuing mechanism is an effective mechanism for favoring citizens with “low time costs.”[1]

Examples of goods that have been allocated in developing countries using queuing include rice in Sri Lanka (Gunawardana, 2000), banking services in Nigeria (Woldie, 2003), formal sector jobs in Nicaragua (Pisani & Pagan, 2003), and jobs in general in urban Ethiopia (Serneels, 2007).[2] A salient attribute of these goods allocation mechanisms is that they involve *waiting* in

* Corresponding author.
E-mail addresses: aabgsh@rit.edu (A.A. Batabyal), deangg@rpi.edu (G.J. DeAngelo).

[1] See Lui (1985), Lee and Tollison (2009), and the many references cited in these two papers for a more elaborate discussion of these ideas.

[2] An example of another good that has been allocated using queuing mechanisms is gasoline. See Deacon and Sonstelie (1989) for more details. In addition, as Laih and Chen (2009) and Munisamy (2010) point out, services such as port facilities and timber terminals have also been allocated by queuing.

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line by citizens. In other words, citizens have to actually wait in queue to obtain the good that is being allocated by one or more public officials.

These queuing mechanisms take on particular significance during times of acute scarcity brought on either by natural factors or by governmental policies. Batabyal (2005) tells us that in such difficult times, the available supplies of the relevant scarce goods are allocated by public officials in many developing countries in particular using queuing mechanisms of one sort or another. From our standpoint, what is noteworthy is that in times of acute scarcity, demand for a good greatly exceeds its supply. Therefore, if a good for which there is excess demand is to be allocated to citizens by means of a queuing mechanism then, in the time period of interest, it is important for a public provider to think carefully about both the *length* of the queue and the *number* of citizens who are turned away without being provided the good in question. To see this clearly, consider the following three examples from three different nations.

Venezuela's President Hugo Chavez has been very keen to bring the powerful state owned oil company *Petroleos de Venezuela* (PDVSA) under his control. This desire has caused considerable political unrest. As reported by the Economist (Anonymous, 2003, 2009), there was a severe shortage of both cooking gas and petrol in the nation. As a result, in some provider facilities, there were queues of citizens that could be measured not in hours but in days. This state of affairs gave rise not only to a secondary market for places in the fuel queues but it also resulted in considerable economic turmoil and violence.

Former Egyptian President Hosni Mubarak's governmental policies made life difficult for many of this nation's 75 million citizens. Specifically, the prices of many food items rose and it became routine for public officials to allocate scarce goods such as bread by means of queuing mechanisms. The Economist (Anonymous, 2008) delineates situations where, *inter alia*, rising food prices led to spontaneous protests and to urban riots that led, as in Venezuela, to violence.

Until recently, energy shortages of every kind plagued Iraq. As reported by Vick (2004), gas lines in Baghdad often reached new lengths and this unfortunate state of affairs created yet another venue for violence. Vick (2004) notes in his story that at least two people were killed in Baghdad over their place in line or over allegations of watering down the goods. These three examples[3] help explain why it is necessary for a public provider, who utilizes a queuing mechanism to allocate a particular good, to be sensitive about both the *length* of the pertinent queue and about the *number* of citizens who are turned away without being provided the good that is being allocated with a queuing mechanism. Such sensitivity can diminish the likelihood of violence either by lowering overly long citizen wait times or by reducing the number of citizens who return home without receiving the scarce good in question.

In spite of the demonstrated salience of accounting for queue length and the number of citizens who are not provided a good that is allocated with a queuing mechanism, to the best of our knowledge, these two issues have received scant theoretical attention in the extant literature. Lui (1985), Batabyal and Nijkamp (2004), and Batabyal and Yoo (2007) have used queuing models to analyze bribery and corruption. Athreya and Majumdar (2005) have studied a queuing model in which a reorganization of two government departments potentially increases efficiency. Although all four of these papers have shed light on the properties of alternate queuing mechanisms, they have *not* examined the violence issue either from the standpoint of regulating the queue length or form the standpoint of reducing the number of citizens who end up not being provided a good that is rationed with a queuing mechanism.

Recently, Batabyal (2005) and Batabyal and Herath (2010) have shed light on the topic of violence prevention in the context of goods allocation with queuing mechanisms. Batabyal (2005) has used the M/M/1 queuing model—see Ross (1996, pp. 254–255) or Tijms (2003, pp. 188–190) for textbook treatments and Batabyal and Nijkamp (2008) for an alternate application—to ascertain the smallest capacity that will keep the probability of violence below an exogenously given value. Batabyal and Herath (2010) have utilized queuing models to compute the long run delay per citizen in allocating a good and the long run fraction of time that a good allocating public official is busy.

In this paper, we provide a different stochastic perspective on goods allocation with queuing and the occurrence of violence. Relative to the prior contributions of Batabyal (2005) and Batabyal and Herath (2010), the use of this paper's perspective allows us to (i) shed light on questions that are different from those studied previously, (ii) provide an “accuracy check” on the calculated values of our model's so called equilibrium probabilities, and (iii) demonstrate the use of different modeling approaches to questions concerning goods allocation by queuing in the possible presence of violence. Specifically, we use the theory of discrete-time Markov chains (DTMCs) to construct and analyze models in which we explicitly account for both queue length and the number of citizens who are not provided a good that is allocated with a queuing mechanism.[4] We first delineate a version of our DTMC model in which there is no capacity constraint. No capacity constraint means that the public good allocation facility's waiting room is arbitrarily large and hence there is no upper limit on the number of citizens who can be asked to wait in this room. This point is explained in greater detail in Sections 2.1 and 3.1 below. Second, we specify two key properties of this

[3] See Glenny (2008) and Lee and Tollison (2009) for more such examples of violence in the context of resource allocation mechanisms involving queuing.

[4] A discrete-time Markov chain (DTMC) is an example of a stochastic process that takes on a finite or a countable number of possible values. This set of possible values of the stochastic process is typically denoted by the set of non-negative integers. The key attribute of a Markov chain is that the conditional probability distribution of any future state, given the past states and the present state, is independent of the past states and depends only on the present state. In addition, this Markov chain is said to be discrete-time because transitions between states occur at fixed times only. Finally, for a DTMC, the time spent by this stochastic process in any one state before it makes a transition to some other state is one time unit long. DTMCs have found wide applicability in biology, computer science, economics, and operations research. Within economics, DTMCs have been used to study problems in, *inter alia*, agriculture (see Batabyal & Beladi, 2004), industrial organization (see Tirole, 1988), and natural resource economics (see Rohlin & Batabyal, 2005). See Ross (1996, pp. 163-230) or Tijms (2003, pp. 81-139) for lucid textbook accounts of DTMCs.

model and then we derive our first metric of the likelihood of violence. Finally, we describe an alternate version of our DTMC model with a capacity constraint and then we derive our second metric of the likelihood of violence.

The rest of this paper is organized as follows. Section 2.1 delineates our DTMC theoretic model of a scarce good allocation process with no capacity constraint. Section 2.2 first presents two key results for the DTMC model described in Section 2.1 and then this section derives our first metric of the likelihood of violence. Next, Section 3.1 describes a DTMC theoretic model of a good allocation process with a capacity constraint. Section 3.2 determines the long run fraction of citizens who are not provided the scarce good in question. Finally, Section 4 concludes and then discusses two ways in which the analysis in this paper might be extended.

2. Good allocation without a capacity constraint

2.1. Preliminaries

Consider a particular region in a developing country or in a transition economy in a time of acute economic scarcity. Within this region, we focus on $c \in \mathbb{N}$ public officials who are in charge of a facility that is responsible for allocating an essential, scarce good such as a loaf of bread or a canister of gasoline to citizens. Here, $\mathbb{N}$ refers to the set of positive integers. Citizens arrive at the public good allocation facility in accordance with a stationary Poisson process with rate $\lambda > 0$. These arriving citizens are asked to wait temporarily in a waiting room with no capacity constraint.

At fixed clock times $t = 0,1,2,\ldots$ citizens are moved out of the waiting room and provided the pertinent scarce good. Each public official allocates the scarce good to one citizen at a time. We suppose that it takes any one of the c public officials one time slot to allocate the scarce good to the waiting citizens. Further, the c public officials perform their tasks in a way so that the allocation of the relevant good to citizens can only start at the clock times $t = 0,1,2,\ldots$ To keep the underlying model from becoming unstable, we assume that $\lambda < c$. In words, the arrival rate of the citizens is less than the good allocation capacity of the public officials under study.[5]

To analyze the above description of events with a DTMC model, let us define the random variable X_n to be equal to the number of citizens in the public facility's waiting room—excluding any citizens who are being allocated the scarce good—just prior to the clock time $t = n$. Then, some careful thought and our knowledge of the properties of DTMCs leads us to

Proposition 1. *The stochastic process* $\{X_n, n = 0, 1, 2\ldots\}$ *is a DTMC with infinite state space* $I = \{0, 1, 2, \ldots\}$.

To specify the one-step transition probabilities p_{ij} of our DTMC model with no capacity constraint, it is helpful to partition the state space into two parts. For the first part, we have $0 \le i < c$ and $j = 0, 1, 2,\ldots$ For the second part, we have $i \ge c$ and $j = i - c, i - c + 1,\ldots$ Now, putting these two pieces of information together, the one-step transition probabilities we seek are given by[6]

$$p_{ij} = \frac{e^{-\lambda}\lambda^j}{j!}, 0 \le i < c, j = 0, 1, 2, \ldots \tag{1}$$

and

$$p_{ij} = \frac{e^{-\lambda}\lambda^{j-i+c}}{(j-i+c)!}, i \ge c, j = i-c, i-c+1, \ldots \tag{2}$$

Our stability supposition ($\lambda < c$) implies that the DTMC under study $\{X_n, n = 0, 1, 2\ldots\}$ satisfies Assumption 3.3.1 in Tijms (2003, p. 98). In turn, this means that our DTMC model possesses equilibrium probabilities π_j, $j = 0, 1, 2,\ldots$ Specifically, these equilibrium probabilities are the unique solution to the equilibrium equations

$$\pi_j = e^{-\lambda}\frac{\lambda^j}{j!}\sum_{k=0}^{c-1}\pi_k + \sum_{k=c}^{c+i} e^{-\lambda}\frac{\lambda^{j-k+c}}{(j-k+c)!}\pi_k, j = 0, 1, 2, \ldots \tag{3}$$

in conjunction with the normalizing equation $\sum_{j=0}^{\infty}\pi_j = 1$. In words, the equilibrium probability π_j is the long run probability that there are j citizens in the good allocation facility's waiting room. This completes the discussion of our DTMC model of the good allocation process by c public officials to arriving citizens with no capacity constraint. Before proceeding further, we note that the aforementioned equilibrium probabilities are also known as "long run," "limiting," and as "steady state" probabilities. As noted in Tijms (2003, pp. 96–106), the reader ought to understand that it is standard to refer to these probabilities as equilibrium probabilities. Our next task is to specify two key properties of this model and then we derive our first metric of the likelihood of violence.

[5] There is some similarity between the way in which we are modeling the good allocation problem and the occurrence of queuing at signalized intersections.

[6] Eqs. (1) and (2) follow from the facts that (i) we are studying a DTMC, (ii) a DTMC is history independent, and (iii) the times between successive arrivals for a stationary Poisson process are exponentially distributed and the exponential distribution is memoryless. See Tijms (2003, pp. 114-115) for a textbook description of the specification of similar one-step transition probabilities.

2.2. Two properties and the likelihood of violence

The first property we focus on concerns the relationship between the Poisson arrival rate of the citizens $\lambda > 0$ and the long run number of citizens who pass through the good allocation facility under study. Note that we can also think of this relationship as a relationship between two averages. The first average is the average input which describes the arrival rate of the citizens to the good allocation facility. Obviously, this average is given by λ. The second average which describes the long run mean number of citizens who pass through the good allocation facility is given by what is known in the queuing theory literature as the average throughput. Now, using standard techniques—see Tijms (2003, pp. 96-118)—and the equilibrium probabilities described in Section 2.1, we are led to

Proposition 2. *The average throughput of our good allocation facility is* $\sum_{j=1}^{c-1} j\pi_j + c\sum_{j=c}^{\infty}\pi_j$.

Some thought ought to convince the reader that in order for our DTMC model of good allocation by public officials to be tractable, the average input into the model must equal the average throughput. This means that, mathematically, we must have

$$\lambda = \sum_{j=1}^{c-1} j\pi_j + c\sum_{j=c}^{\infty}\pi_j. \tag{4}$$

The first property that we seek is given by Eq. (4) and we now state this property in

Proposition 3. *In our DTMC model of good allocation by c public officials, the mean arrival rate of the citizens to the public facility equals the long run average number of citizens who pass through this facility.*

In a given practical setting, we would be interested in calculating our DTMC model's equilibrium probabilities π_j, $j = 0, 1, 2,...$ A natural question that arises when undertaking such a task pertains to the accuracy of the calculations. In this regard, it is helpful to note that the property described by Eq. (4) and stated in Proposition 3 can be used as an "accuracy check" on the calculated values of the equilibrium probabilities. We now proceed to the second property of our DTMC model.

This second property concerns the derivation of a closed-form expression for the long run average number of citizens who are present in the waiting room of the good allocation facility under study. Let us denote this average by *LRAC*. Suppose that a cost at rate k is incurred by our public facility when there are k citizens in the waiting room for $k = 0,1,2,\ldots$ In addition, suppose that the public officials inspect the waiting room in their facility every T time units to determine the number of people who are actually waiting. Only at these inspection time points are the waiting citizens assigned to free public officials.[7] Now, the important point to grasp is that the *LRAC* metric we seek is given by the long run average cost per unit time incurred by our good allocation facility. In other words, given the above cost structure, there exists a *definite relationship* between the *long run average number of citizens* who are present in the waiting room and the *long run average cost* as specified above. So, if we can compute this average cost metric then we will, in effect, have also computed the "average number of citizens waiting" metric because these two metrics are equal to each other.[8]

Recall that citizens arrive at our good allocation facility in accordance with a Poisson process with rate $\lambda > 0$. Therefore, to derive the above mentioned *LRAC*, we can make use theorem 1.1.3 in Tijms (2003, p. 6). Applying this theorem to our problem, we reason that the average cost incurred in a time slot given that j citizens are present in the waiting room at the beginning of this time slot is

$$(j - j_c)T + \frac{1}{2}\lambda T^2, \tag{5}$$

where costs are associated with the effective number of waiting citizens given by $(j - j_c)$.

Now, to compute the long run average cost per time slot, we use Theorem 3.3.3 in Tijms (2003, p. 103). The application of this ergodic theorem to our problem and Eq. (5) tell us that the desired average cost is given by

$$\sum_{j=0}^{\infty}\left[(j - j_c)T + \frac{1}{2}\lambda T^2\right]\pi_j = \sum_{j=c}^{\infty}(j - c)T\pi_j + \frac{1}{2}\lambda T^2, \tag{6}$$

[7] In Section 2.1 we noted that at fixed clock times $t = 0,1,2,\ldots$ citizens are moved out of the waiting room and provided the relevant scarce good. This means that at intervals of length one time unit, citizens are moved out of the waiting room. Now we are saying that the waiting room is inspected every T time units and only at these inspection points are waiting citizens assigned to free public officials. This may seem like temporal metrics whose lengths are dissimilar are being used in the derivation of the *LRAC* criterion. However, note that as pointed out in the body of the paper right after Eq. (6), the expression in this equation, i.e., in (6), is divided by T to give us the expression for the *LRAC* criterion stated in Proposition 4.

[8] The theoretical justification for this equality arises from the existence of a so called "ergodic theorem" (see Theorem 3.3.3 in Tijms (2003, p. 103)). This theorem shows how to compute the long run average cost per unit time in terms of a DTMC's equilibrium probabilities.

Dividing the RHS of Eq. (6) by T gives us an expression for the long run average cost per unit time and, as discussed previously in this section, this expression is also equal to the long run average number of citizens in our good allocation facility or *LRAC*. Performing the division, we get

Proposition 4. *The long run average number of citizens in our public facility is given by* $LRAC=\sum_{j=c}^{\infty}(j-c)\pi_j+\frac{1}{2}\lambda T$.

From Proposition 4, it is clear that the long run average number of citizens in our public facility's waiting room is a function of, *inter alia*, the Poisson citizen arrival rate $\lambda>0$ and the inspection time variable T. It is unlikely that the public officials under study will be able to control the rate $\lambda>0$ at which citizens arrive at the good allocation facility. This tells us that if our public officials staffing the facility would like to reduce crowding in their waiting room then they will want to *reduce T* or, equivalently, *increase* the frequency with which they inspect the waiting room before assigning citizens to each of them for the allocation of the pertinent scarce good. We now proceed to the derivation of our first metric of the likelihood of violence.

At a good allocation facility of the sort studied in this paper, violence may occur but it clearly does not have to occur. In other words, the occurrence of violence is a *probabilistic* event. This said, the discussion in Section 1 of this paper suggests that violence often occurs when citizens have to wait a long time in queue before they are allocated the scarce good in question. Therefore, one reasonable approach to studying the likelihood of violence would be to focus on the *long run average delay* that citizens experience in the public facility's waiting room. Let *LRAD* denote this long run average delay per citizen. To obtain a closed-form expression for this metric, we make use of Little's formula—see Ross (1996, pp. 139–140) or Tijms (2003, pp. 50–52) for more details—from the queuing theory literature. Applied to our problem, Little's formula tells us that the *LRAD* is given by the long run average number of citizens in our good allocation facility's waiting room or *LRAC* divided by the Poisson arrival rate of the various citizens $\lambda>0$. Therefore, dividing the *LRAC* expression in Proposition 4 by $\lambda>0$ gives us our first metric of the likelihood of violence. We now state this finding as

Proposition 5. *The long run average delay per citizen in our good allocation facility's waiting room is* $LRAD=\left\{\sum_{j=c}^{\infty}(j-c)\pi_j\right\}/\lambda+(1/2)T$.

Inspecting Proposition 5, we see that *unlike* the expression for LRAC given in Proposition 4 where the *direct* effect of the Poisson citizen arrival rate $\lambda>0$ was multiplicative, now this direct effect of λ is divisive in the sense that it appears in the denominator of the first ratio describing the LRAD. In contrast and *like* the expression for LRAC in Proposition 4, LRAD is an *increasing* function of the inspection time variable T. Therefore, if the public officials staffing the good allocation facility would like to reduce the likelihood of violence by citizens emanating from long wait related delays then they ought to *lessen* T or *raise* the frequency with which they inspect the waiting room before assigning citizens to each of them for the allocation of the scarce good.

In the models of this section, we do *not* have a capacity constraint for the number of queuing citizens in our good allocation facility's waiting room. However, it is clear that all waiting rooms associated with good allocation facilities can accommodate only a finite number of citizens and hence, realistically speaking, it is essential to explicitly account for a capacity constraint. This is what we now do in the next section of this paper.

3. Good allocation with a capacity constraint

3.1. Preliminaries

The probabilistic environment in which our c public officials allocate the scarce good in question to arriving citizens is the same as in Section 2.1. The only difference is that we now take a different approach to studying the likelihood of violence by introducing an explicit constraint that our public officials apply to effectively cap the total number of citizens who queue in the good allocation facility's waiting room.

In particular, we suppose that once K citizens are in the waiting room, no more citizens are allowed into the good allocation facility. Citizens who arrive at this facility when the waiting room is full, i.e., when K citizens are already present, are turned away and they leave without obtaining the scarce good under study. In this setting, our first task now is to modify the Section 2.1 one-step transition probabilities of the DTMC $\{X_n, n=0,1,2\ldots\}$ describing the number of citizens in the public facility's waiting room prior to the various clock times.

The state space now is not the infinite state space $I=\{0,1,2,\ldots\}$ of Section 2.1. Instead, the pertinent state space is $I=\{0,1,2,\ldots,K\}$. This change has noticeable implications for our DTMC model's one-step transition probabilities given in Eqs. (1) and (2). Some thought tells us that the modified one-step transition probabilities are

$$P_{IK}=\sum_{l=K}^{\infty}\frac{e^{-\lambda}\lambda^l}{l!},0\le i<c,P_{iK}=\sum_{l=K-(i-c)}^{\infty}\frac{e^{-\lambda}\lambda^l}{l!},i\ge c,\text{and } p_{ij}=0,j>K. \tag{7}$$

Even with the capacity constraint K in place, violence in our good allocation facility may still occur if "too many citizens" are turned away from the facility without being provided the scarce good in question. Now, what constitutes "too many citizens" will obviously depend on the specifics of individual settings. Even so, we contend that this concept of "too many citizens" and hence

the likelihood of violence is very closely related to the long run *fraction* of all citizens who are turned away from the public facility without being provided the scarce good.[9] Therefore, in our second approach to the likelihood of violence, we compute a closed-form expression for this fraction.

3.2. Long run fraction of citizens who are turned away

Let us denote the long run fraction we seek by *LRFC*. Since we are conducting our analysis from a long run perspective, we will, as in Section 2.2, once again need to work with the relevant equilibrium probabilities. To this end, let $\{\pi_j^{(K)}, j-0, 1, 2, \ldots K\}$ denote the equilibrium probability distribution of our DTMC model of the scarce good allocation process that we have been studying thus far. Our next task is to compute the long run mean number of citizens who pass through the good allocation facility.

Recall from Section 2.2 that this long run mean is our model's average throughput. Modifying the throughput expression in Proposition 2 to account for the capacity constraint K, our new expression for the DTMC model's average throughput is $\sum_{j-1}^{c-1} j\pi_j^{(K)} + c\sum_{j=c}^{K}\pi_j^{(K)}$. Now, the average number of citizens who are turned away from the good allocation facility and hence not provided the scarce good per unit time equals the difference between the average input λ and the average throughput $\sum_{j=1}^{c-1} j\pi_j^{(K)} + c\sum_{j=c}^{K}\pi_j^{(K)}$. Therefore, this average is given by $\lambda-\sum_{j=1}^{c-1} j\pi_j^{(K)} + c\sum_{j=c}^{K}\pi_j^{(K)}$. Finally, dividing the preceding expression by the Poisson citizen arrival rate λ gives us a closed-form expression for our second metric of the likelihood of violence. We state our result in

Proposition 6. *The long run fraction of all citizens who are not provided the scarce good or LRFC* $= (1/\lambda)\left[\lambda-\sum_{j=1}^{c-1} j\pi_j^{(K)} - c\sum_{j=c}^{K}\pi_j^{(K)}\right]$.

Note that the long run fraction in Proposition 6 can also be thought of as a long run loss probability. Inspecting Proposition 6, we see that the *magnitude* of *LRFC* depends on the magnitude of the two "fraction reducing" terms $(1/\lambda)\sum_{j=1}^{c-1} j\pi_j^{(K)}$ and $(c/\lambda)\sum_{j=c}^{K}\pi_j^{(K)}$. In turn, the magnitude of these two "fraction reducing" terms depends on the number of public officials c and the capacity constraint K. Therefore, consistent with our intuition, when either c or K is large, the magnitudes of one or both the "fraction reducing" terms is also large and hence the long run fraction of all citizens who are not provided the good is small.

More generally, Proposition 6 tells us that when the focus of our public officials is on reducing the likelihood of violence in their good allocation facility, the long run fraction of citizens who will come away from this facility without the scarce good is equal to the average number of citizens who are turned away divided by the parameter or the rate of the Poisson arrival process of the various citizens. Examining Proposition 6, it is straightforward to confirm four comparative statics results.

First, the *larger* the individual state equilibrium probabilities, i.e., the π_j^is, the *smaller* is the long run fraction of citizens who are turned away without being provided the pertinent scarce good. Second, looking at the *direct* effect only of the rate λ on *LRFC*, the *quicker* the rate at which citizens arrive in our good allocation facility, the *larger* is the fraction of citizens who leave the facility without the scarce good in question. Third and consistent with a previous observation of ours, as the number c of public officials allocating the scarce good *increases*, the long run fraction of turned away citizens *decreases*. Finally, if we *raise* the capacity constraint K then the long run fraction of citizens who leave the facility without the scarce good also *declines*. This completes our discussion of good allocation with a capacity constraint.

4. Conclusions

In this paper, we used the theory of DTMCs to construct and analyze a queuing based good allocation process with the possibility of violence. In this setting, we explicitly accounted for both the queue length and the number of citizens who are not provided the good under study. We described a version of our DTMC model in which there is no capacity constraint. Next, we stated two key properties of this model and then we derived our first metric of the likelihood of violence. Finally, we delineated an alternate version of our DTMC model with a capacity constraint and then we derived our second metric of the likelihood of violence.

The analysis in this paper can be extended in a number of different directions. In what follows, we suggest two potential extensions. First, consistent with the discussion in the last paragraph of Section 3.2, it would be useful to set up and solve an optimization problem for our public officials in which these officials select the capacity constraint K to, for instance, optimize the net social benefit to the region under study from the allocation of the pertinent scarce good.

Second, let K(α) be the smallest value of the capacity constraint K for which $LRFC \leq \alpha$ for a given value of α. Let $\theta = \lambda/c$ and suppose that $\theta = 0.90$, 0.95 and $c = 1$, 5, 10. Then, for these and other values of θ and c, we could compute actual values of $K(a)$. This will enable us to answer questions such as whether $K(\alpha)$ does or does not increase logarithmically in α as α increases. This will also allow us to shed light on the asymptotic behavior of *LRFC* as K becomes large. Studies that analyze these aspects of the underlying problem will enhance our understanding of the connections between queue based good allocation processes and the occurrence of violence.

[9] Real world instances of violence stemming from the capacity of the waiting room are described in Allen (2009) in medical settings and in http://www.havanatimes.org/?p=39328 for various settings in Cuba. This website was accessed on 31 October 2011.

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